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On the Q^2 dependence of the measured polarized structure functions

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Abstract

We analyse the available data on the polarized asymmetries A_1 for proton, neutron and deuteron targets. We use a homogeneous and updated set of unpolarized structure functions to derive g_1 from A_1, and we accurately correct for the scaling violations in order to obtain $g_1(x, Q^2)$ with the same Q^2 for all x values. The contribution to the Q^2 evolution of a possible large gluon polarized density is also considered. The implications for the Ellis-Jaffe and for the Bjorken sum rules are discussed.

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New data on the polarized structure functions g_1 of deuterium nuclei[1] and of neutrons[2] have recently been published, which add to previous data on g_1 of protons[3,4]. Of special importance for the physical interpretation of the results is the derivation from the data of the first moments related to the Ellis-Jaffe[5] and the Bjorken[6] sum rules. While the data on $g_1(x, Q^2)$ are collected at different values of Q^2 for different values of x, the evaluation of moments in x requires the same value of Q^2 for all values of x. Thus, for a correct evaluation of moments, in principle, one must apply corrections to the data points in order to take the Q^2 evolution into account and reduce each data point in x to a common Q^2 value. As a first approximation to the solution of this problem one can imagine to take advantage of the fact that, within the present accuracy of the data, the primary measured quantity, the asymmetry

$$A_1 = \frac{\sigma^{\uparrow\downarrow} - \sigma^{\uparrow\uparrow}}{\sigma^{\uparrow\downarrow} + \sigma^{\uparrow\uparrow}}, \tag{1}$$

shows no appreciable Q^2 dependence[1−4]. Then, $g_1(x, Q^2)$ at fixed Q^2 for all x values is obtained from $A_1(x)$ through the relation $A_1 = g_1/F_1$, with F_1 being the unpolarized structure function obtained from a fit of existing data. This procedure has been used in refs. [7,8]. As confirmed by these approximate analyses, the resulting corrections to the first moments are small in comparison with the present experimental errors. However, in view of more precise forthcoming data, it is interesting to collect the known results on the Q^2 evolution of polarized parton densities[9,10] and describe a more accurate method for reducing the set of data to the same Q^2 value for all x bins. In fact, an obvious shortcoming of the approximation of refs. [7,8] is that the asymmetry A_1 is not predicted to be a constant in Q^2 by the correct evolution equations, so that, by assuming it to be a constant, one makes an error of the same order as the effect under study. As the whole Q^2 correction is small with respect to the present experimental errors, it is no surprise that the variation of A_1 with Q^2 is not discernible in the present data.

We describe in the following a more correct procedure that starts from $A_1(x_i, Q_i^2)$, for the i-th experimental point, constructs $g_1(x_i, Q_i^2) = A_1(x_i, Q_i^2)F_1(x_i, Q_i^2)$ and finally evolves $g_1(x_i, Q_i^2)$ into $g_1(x_i, Q_0^2)$, with Q_0^2 being a suitable common value, for example the average Q^2 value in the experiment. In passing, we explicitly compute the predicted Q^2 dependence of the asymmetry A_1 for p, n and d targets, and check that the available data are indeed compatible with it. The Q^2 evolution of polarized quark densities also depends on the gluon polarized density. We compute the evolution for two limiting cases. In the first case we assume a negligible amount of polarized

gluons. In the second case we start from a polarized gluon density large enough to entirely explain in terms of gluons (with neglible polarized strange sea) the violation of the Ellis-Jaffe sum rule observed by the EMC experiment on protons, according to the mechanism based on the anomaly proposed in refs. [11,12] and further discussed in refs. [13–15].

We consider the polarized structure functions

$$g_1^p = \frac{1}{2}\left[\frac{4}{9}\Delta u + \frac{1}{9}\Delta d + \frac{1}{9}\Delta s\right] \tag{2}$$

$$g_1^n = \frac{1}{2}\left[\frac{4}{9}\Delta d + \frac{1}{9}\Delta u + \frac{1}{9}\Delta s\right], \tag{3}$$

where

$$\Delta q = q_+ + \bar{q}_+ - q_- - \bar{q}_- \tag{4}$$

and q_+ (q_-) is the density of the quark q with positive (negative) helicity in a proton with positive helicity. We recall that, according to the mechanism discussed in refs. [11–15], the effective polarized quark densities defined by eqs. (2) and (3) are split into a part conserved by the two-loop Q^2 evolution equations, which is expected to be closely related to constituent quarks, and the gluon component, which, due to the anomaly, does not decouple from the first moment at large Q^2.

We have

$$g_1^p = \quad g_1^{(3)} + g_1^{(8)} + g_1^{(S)} \tag{5}$$

$$g_1^n = -g_1^{(3)} + g_1^{(8)} + g_1^{(S)}, \tag{6}$$

where

$$g_1^{(3)} = \frac{1}{12}\left[\Delta u - \Delta d\right] \tag{7}$$

$$g_1^{(8)} = \frac{1}{36}\left[\Delta u + \Delta d - 2\Delta s\right] \tag{8}$$

$$g_1^{(S)} = \frac{1}{9}\left[\Delta u + \Delta d + \Delta s\right]. \tag{9}$$

The evolution equation for Δq is given by [9]

$$Q^2\frac{d}{dQ^2}\Delta q = \frac{\alpha_S(Q^2)}{2\pi}\left[\Delta q \otimes \Delta P_{qq} + 2\Delta g \otimes \Delta P_{qg}\right]. \tag{10}$$

Here $\Delta g = g_+ - g_-$, g is the gluon density function, and

$$\Delta P_{qq}(z) = \frac{4}{3}\left(\frac{1+z^2}{1-z}\right)_+ \tag{11}$$

$$\Delta P_{qg}(z) = \frac{z^2-(1-z)^2}{2}. \tag{12}$$

The convolution product $\otimes$ is defined as usual by

$$(f\otimes g)(x) = \int_x^1 \frac{dz}{z} f(z) g\left(\frac{x}{z}\right). \tag{13}$$

Combining eqs. (2), (3) and (10), we find

$$Q^2\frac{d}{dQ^2}g_1^{p,n} = \frac{\alpha_S(Q^2)}{2\pi}\left[g_1^{p,n}\otimes\Delta P_{qq} + \frac{2}{3}\Delta g\otimes\Delta P_{qg}\right], \tag{14}$$

or

$$dg_1^{p,n} = \frac{\alpha_S(Q^2)}{2\pi} d\log\frac{Q^2}{\Lambda^2}\left[g_1^{p,n}\otimes\Delta P_{qq} + \frac{2}{3}\Delta g\otimes\Delta P_{qg}\right]. \tag{15}$$

This equation can be approximately integrated by observing that

$$\frac{\alpha_S(Q^2)}{2\pi} d\log\frac{Q^2}{\Lambda^2} = -\frac{1}{2\pi b} d\log\alpha_S(Q^2) + \mathcal{O}(\alpha_S^2), \tag{16}$$

where $2\pi b = (33-2n_f)/6 = 9/2$. For a small displacement in Q^2, we can neglect the Q^2 dependence of $g_1^{p,n}$ and Δg on the r.h.s. of eq. (15), and obtain

$$g_1^{p,n}(x,Q^2) - g_1^{p,n}(x,Q_0^2) = \frac{2}{9}\log\frac{\alpha_S(Q_0^2)}{\alpha_S(Q^2)}\left[g_1^{p,n}\otimes\Delta P_{qq} + \frac{2}{3}\Delta g\otimes\Delta P_{qg}\right] + \mathcal{O}(\alpha_S^2). \tag{17}$$

Actually, in the numerical computations, we use the two-loop expression of $\alpha_S(Q^2)$ in terms of Λ. We take $\Lambda_{\overline{MS}}(n_f=3) = 383^{+126}_{-116}$ MeV, which corresponds to $\alpha_S(m_Z^2) = 0.118 \pm 0.007$. With the definitions given above, taking into account the definition of the + distribution, one obtains

$$g_1^{p,n}\otimes\Delta P_{qq} = \frac{4}{3}\int_x^1 \frac{dz}{z}\frac{1+z^2}{1-z}\left[\frac{1}{z}g_1^{p,n}\left(\frac{x}{z}\right) - g_1^{p,n}(x)\right] + \frac{4}{3}\left[x + \frac{x^2}{2} + 2\log(1-x)\right] \tag{18}$$

and

$$\Delta g\otimes\Delta P_{qg} = \int_x^1 \frac{dz}{z}\frac{z^2-(1-z)^2}{2}\Delta g\left(\frac{x}{z}\right). \tag{19}$$

An extreme parametrization of $\Delta g(x)$ has been proposed in ref. [14] for the EMC experiment:

$$\Delta g(x) = C x^{-0.3}(1-x)^7, \tag{20}$$

with the normalization C such that $\int_0^1 \Delta g(x)dx = 5$. In the following, for each experiment we use the same form for $\Delta g(x)$, but with a rescaled normalization given by

$$\int_0^1 \Delta g(x)dx = 5\frac{\alpha_S(Q^2_{EMC})}{\alpha_S(Q^2_{EXP})}, \tag{21}$$

where $Q^2_{EMC} = 10.7$ GeV^2 and Q^2_{EXP} are the average values of Q^2 for the EMC and for the experiment under consideration, respectively. This amount of polarized gluons is extreme in the sense that all the EMC deviation from the Ellis-Jaffe sum rule is attributed to the effect of gluons, while some amount of polarized strange sea is indeed plausible. Also, the above polarized gluon distribution nearly saturates the positivity bound from the known unpolarized gluon density especially at large x [14].

In the original treatment of the data[1–4], in order to obtain the values of g_1, the measured asymmetry $A_1(x)$ is multiplied by $F_1(x, Q_0^2)$, where Q_0^2 is the average Q^2 of the experiment. In the present context, obviously we cannot tolerate this neglect of an important contribution to the Q^2 dependence. As already stated in the introduction, we construct $g_1(x_i, Q_i^2) = A_1(x_i, Q_i^2)F_1(x_i, Q_i^2)$. At this stage we can implement, for all experiments, an updated form for the unpolarized structure functions that takes also the recent measurements by the NMC[16] into account. For this purpose we use the fit to the structure function F_2 from ref. [16] and the fit to $R = F_2/(2xF_1) - 1$ from ref. [17]. Starting from the resulting values of $g_1(x_i, Q_i^2)$, we evolve each point up to $g_1(x_i, Q_0^2)$. For the evolution we use a fit to the data points $g_1(x_i, Q_i^2)$ as an input for $g_1(x)$ in the r.h.s. of eq. (17) and consider both the cases with and without gluons. The modified experimental points for g_1, and the corresponding ones corrected for the evolution to Q_0^2, chosen as the average Q^2 of the experiment, are shown in fig. 1. In the same figure we also show a fit to the data, (solid line), corrected for the evolution with $\Delta g = 0$. Our fits are given by

$$g_1^p(x, Q_0^2) = x^{0.2}(1-x)^3(0.589 + 1.07x - 1.29x^2 - 0.792x^3) \tag{22}$$

$$g_1^d(x, Q_0^2) = x^{0.2}(1-x)^6(-0.670 + 15.0x - 46.0x^2 + 43.4x^3) \tag{23}$$

$$g_1^n(x, Q_0^2) = x^{0.2}(1-x)^6(-0.358 - 1.10x + 12.7x^2 - 16.5x^3). \tag{24}$$

By d we actually mean $(p+n)/2$, as the corresponding nuclear correction factor has been taken into account[1]. The extrapolation at low x outside the measured range is also shown, based on fitting only the last two measured points by $Bx^{0.2}$ (i.e. the same α as in the previous fit). Similarly the extrapolation at large x, based on $D(1-x)^3$ fitted on only the last measured point, is also displayed.

In fig. 2 we compare the computed Q^2 dependence of the measured asymmetry for protons with the EMC data[4]. For each bin in x the two curves refer to the computed Q^2 dependence at the lowest and highest point in x. The Q^2 dependence of $A_1(x, Q^2)$ is evaluated by using the evolution equations for $g_1(x, Q^2)$ according to the above formalism and the already mentioned (Q^2-dependent) fit to the unpolarized structure functions in order to obtain $F_1(x, Q^2)$. The results with and without gluons are plotted separately. We see that the difference associated to the gluons in the evolution is small. This is because the evolution equations at x_0 are only sensitive to the parton densities at $x > x_0$: the gluon density, although having a large first moment, being concentrated at small x, has little influence on the evolution in the measured range of x. We confirm that within the present accuracy of the data no evidence for the Q^2 dependence can be seen. But we also see that the effects of the Q^2 dependence cannot be ignored in a more precise analysis. In fig. 3 the predicted Q^2 dependence of the neutron and deuterium asymmetries is shown.

The first moments of g_1 for p, n and d were obtained by three different methods:

- (a) We sum the experimental points bin by bin, excluding the first and the last bins. We then add to this value the integral of the small and large x extrapolations over the remaining x region. This way of treating the end points is safer, since the functions g_1 are rapidly varying there, especially near $x = 1$.
- (b) We integrate our fits of the data over the experimental range excluding the first and the last bins. The remaining region is treated as in (a).
- (c) We simply integrate our fits over the full x range.

The results corresponding to the different integration methods, with and without the corrections for the Q^2 scaling violations, and with and without gluons in the evolution equations, are collected in table 1 for p, n, d and $p - n$, the last being the combination of relevance for testing the Bjorken sum rule. For protons, the EMC[4] and SLAC[3] data have been combined. It can be seen that the differences between moments computed by different methods are well within the errors quoted by the collaborations. We prefer procedure (c) of integrating the overall fit over the complete range. In fact the sum bin by bin does not take the differences in the errors associated with the different bins into proper account, and the extrapolation based on only the last points overemphasizes their significance. Nevertheless, we consider it important to keep in mind the size of the changes from one method to the other.

	I			II			III		
	(a)	(b)	(c)	(a)	(b)	(c)	(a)	(b)	(c)
$I(p)$	0.125	0.122	0.119	0.132	0.126	0.119	0.131	0.127	0.119
$I(d)$	0.029	0.029	0.031	0.034	0.034	0.035	0.031	0.031	0.034
$I(n)$	-0.023	-0.023	-0.022	-0.023	-0.023	-0.023	-0.022	-0.022	-0.020
	Bjorken sum rule								
EMC/SMC	0.192	0.185	0.175	0.196	0.185	0.167	0.200	0.191	0.171
EMC/E142	0.148	0.144	0.141	0.155	0.149	0.141	0.153	0.148	0.140
SMC/E142	0.104	0.104	0.106	0.114	0.114	0.115	0.105	0.105	0.108

Table 1: First moments of g_1 for p, d and n. The group of columns labelled I correspond to uncorrected data, those labelled II (III) to data corrected for evolution with $\Delta g = 0$ (Δg as in eq. (19)). The columns (a), (b) and (c) refer to the different integration methods described in the text.

The results in table 1 are obtained with the central value $\Lambda = 383$ MeV. The effect of varying Λ in the range given above, 267 to 509 MeV, amounts to a relative error of the order of 20 to 40% of the computed correction due to Q^2 evolution. However, because of the smallness of the evolution correction, the absolute effect on the first moments $I(p)$, $I(d)$ and $I(n)$ is always less than ± 0.0012. According to the above discussion, for the first moments $I(p)$, $I(d)$ and $I(n)$ we take the results from columns (c) of table 1. The associated errors are taken to be identical to those quoted in the experimental papers, as is appropriate, given our indicative purposes. We obtain the following results:

$$I(p) = [0.119, 0.119, 0.119] \pm 0.018 \quad (0.126) \tag{25}$$

$$I(d) = [0.031, 0.035, 0.034] \pm 0.027 \quad (0.0245) \tag{26}$$

$$I(n) = [-0.022, -0.023, -0.020] \pm 0.011 \quad (-0.022), \tag{27}$$

where in square brackets we give in a sequence the raw data, the results from the evolution with only quarks, and finally those with quarks and gluons. The values in round brackets are the raw data values quoted by the experimental collaborations (with the error which is also shown). We see that the effect of the Q^2 evolution is small with respect to the ambiguities associated with both the fitting procedure and

the experimental errors.

For the Bjorken difference $I(p) - I(n)$, obtained from the three possible combinations of experiments, we obtain, in the same notation as above:

$$\text{EMC/SMC} \quad : \quad [0.175, 0.167, 0.171] \pm 0.060 \quad (0.200) \tag{28}$$

$$\text{EMC/E142} \quad : \quad [0.141, 0.141, 0.140] \pm 0.022 \quad (0.148) \tag{29}$$

$$\text{SMC/E142} \quad : \quad [0.106, 0.115, 0.108] \pm 0.058 \quad (0.093). \tag{30}$$

Note that the result on the Bjorken sum rule depends on the amount of polarized gluons. One should not be surprised by this effect. It arises because the gluons contribute to correct the individual data points and the average Q^2 is different for each experiment. Also note that the first moments are independent of Q^2 in leading order: once the integral has been computed for a given experiment at Q^2_{EXP}, within the present accuracy, it can be combined with the results of other experiments at different Q^2 values. As a consequence, for each entry in eqs. (28)-(30) we cannot specify the relevant Q^2 within the range defined by the two experiments. In fact, the predicted difference in value for the Bjorken integral is of order $(\alpha_S/\pi)^2$, which is beyond the present accuracy.

In fig. 4 we compare the experimental numbers for the Bjorken sum rules, given in eqs. (28)-(30), with the Q^2-dependent theoretical prediction, including corrections up to order α_S/π or $(\alpha_S/\pi)^3$ calculated in ref. [18]. The higher-twist correction, as computed in ref. [19], is negligible in the Q^2 range shown in fig. 4.

In conclusion, we have studied the effect of scaling violation on the determination of the polarized structure function g_1 for p, n and d targets, and we have proposed a procedure to correct for this effect. Our results for the Bjorken sum rule are summarized in fig. 4. We can see that our results do not differ much from the results quoted by the SMC and E142 collaborations. One important aspect of fig. 4 are the large error bars on the Q^2 value associated with each determination. We have taken these error bars to be the range of the average Q^2 of each pair of experiments. When this uncertainty range is taken into proper account, we see that the experimental results are consistent with the theoretical prediction, although on the low side. Unfortunately, we also see that the result obtained using the E142 data over lap with a region of moderately small Q^2, where the theoretical prediction in strongly unstable. The combination of EMC and SMC data is in a better position from this point of view, but in this case the errors are much larger.

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Figure Captions

Fig. 1: Effect of the Q^2 evolution correction on the data for the structure function $g_1(x, Q^2)$. Both the raw data and the data evolved to the average Q^2 of each experiment (with and without the gluon contribution) are shown. The solid curve is our fit of the data evolved without gluon. The dashed curves represent the extrapolation of the data outside the measured range, as described in the text.

Fig. 2: Computed Q^2 dependence of the proton asymmetry compared with the EMC (squares) and E130-E80 (crosses) data. The low (high) curves correspond to the lower (upper) edge of the x bin.

Fig. 3: Computed Q^2 dependence of the deuteron and neutron asymmetry, with (a) and without (b) the gluon contribution.

Fig. 4: Theoretical prediction for the Bjorken sum rule as a function of Q^2. The dotted curves are obtained using the $\mathcal{O}(\alpha_s)$ formula, for $\Lambda = 267$, 383 and 509 MeV. The solid lines include corrections up to $\mathcal{O}(\alpha_s^3)$, for the same values of Λ. The experimental values are also shown.